$$F = (6/\pi^{3/2})\,\sqrt{B\,t}\,, \qquad (F < 10\%) \qquad (1)$$

$$F = (6/\pi^{3/2})\,\sqrt{B\,t} - (3\,\pi^2)\,B\,t\,, \qquad (F < 85\%) \qquad (2)$$

$$F = 1 - (6/\pi^2)\,\exp(-B\,t) \qquad (F > 85\%) \qquad (3)$$

where $B = \pi^2\,D/R^2$; t = time, D = diffusion constant, R = radius of the sphere.

For two isotopes with indices 3 and 4 we obtain after elimination of time and with $D_3/D_4 = \gamma$:

$$F_3/F_4 = \sqrt{\gamma}\,, \qquad (4)$$

$$F_3/F_4 = (6\,\pi/F_4)\,\{\sqrt{2\,\gamma}\,\sqrt{1 - \pi\,F_4/6 - \sqrt{1 - \pi\,F_4/3}} - \gamma\,(1 - \pi\,F_4/6 - \sqrt{1 - \pi\,F_4/3})\}\,, \qquad (5)$$

$$F_3/F_4 = \{1 - (\pi^2/6)^{\gamma - 1}(1 - F_4)^{\gamma}\}/F_4\,. \qquad (6)$$

The differential ratios of the outgassed isotopes are found to be:

$$\mathrm{d}F_3/\mathrm{d}F_4 = \sqrt{\gamma}\,, \qquad (7)$$

$$\mathrm{d}F_3/\mathrm{d}F_4 = \gamma\{1/\sqrt{1 - \pi\,F_4/3} - 1\} \cdot \{1/(\sqrt{2\,\gamma} \cdot \sqrt{1 - \pi\,F_4/6 - \sqrt{1 - \pi\,F_4/3}}) - 1\}\,, \qquad (8)$$

$$\mathrm{d}F_3/\mathrm{d}F_4 = \gamma\{(\pi^2/6)\,(1 - F_4)\}^{\gamma - 1}\,. \qquad (9)$$

From the Eqs. (7) – (9) the curves shown in Fig. 2 were computed. The error introduced by regarding a sphere instead of a cube is rather small; the term $\pi^2/6$ in Eq. (9) has to be replaced by $\pi^6/512$.

A Unified Interpretation of Mass and Charge Distribution and Prompt Neutron Evaporation in Low-Energy Nuclear Fission

H. O. DENSCHLAG and S. M. QAIM *

Institut für Anorganische Chemie und Kernchemie der Universität Mainz, Germany

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A simple model for a transition state configuration in low-energy fission is given. In its light the latest experimental data on mass and charge distribution and on prompt neutron evaporation are discussed.

The three important characteristics of low-energy nuclear fission, viz, the asymmetry, manifested in the mass distribution, the constant shape and position retained by the heavy mass peak in different fission reactions and the "sawtooth" form of the prompt neutron evaporation curve, have been ascribed to a dumb-bell shape of the compound nucleus immediately before scission [1–3]. The possible effects of the existence of closed shells with 82 neutrons [2], 50 protons or neutrons [4], and 82 neutrons plus 50 protons (= 132 nucleons) [3, 5] in the transition state configuration have also been discussed. Based on these concepts FAISSNER and WILDERMUTH [5] proposed their cluster theory and attempted to give a quantitative description of mass [5] and charge distribution [6] in the fission process. This model, however, has two drawbacks: a) It assumes "equal scission probability anywhere in the neck", an assumption that cannot be confirmed experimentally nor be expected theoretically. b) It postulates subshells at the mass numbers $A = 84$, 90, 96, and 100 in order to reproduce the mass yield curve. These subshells do not have convincing support outside the model.

We believe that the assumption of equal scission probability cannot be maintained and that, in consequence, the mass yield curve cannot be predicted in full from such a simple dumb-bell model. Certain characteristics of the mass yield curve, however, can serve as a check on the validity of the overall picture. A better issue for comparing predictions and experimental findings are the prompt neutron evaporation data and, especially, the charge distribution curve.

Reprint requests to Dr. H. O. DENSCHLAG, Institut für Anorganische Chemie und Kernchemie der Universität Mainz, *D-6500 Mainz*, F. v. Pfeiffer-Weg 14.

* Alexander von Humboldt Fellow, on leave from Pakistan Atomic Energy Commission.

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In the following article, we present a simple dumb-bell picture of the transition state configuration and discuss it in the light of the most recent data on mass and charge distribution [7] and on prompt neutron evaporation [7, 8].

Model

A schematic sketch of the postulated shape of the compound nucleus, ^{236}U, before scission in the asymmetric mode of thermal neutron induced fission is presented in Fig. 1. The larger sphere of the

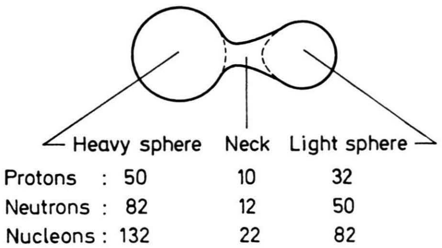

Fig. 1. Schematic sketch of the postulated shape of the compound nucleus before scission.

dumb-bell configuration is assumed to conserve the double magic structure with 50 protons and 82 neutrons and the smaller one to be composed of 50 neutrons and 32 protons. This leaves 12 neutrons and 10 protons in the neck. Fission reactions yielding fragment masses between $A' = 132$ and 154 or 82 and 104 take place exclusively in the neck. Here A' indicates the mass before prompt neutron emission. The point at which scission occurs within the neck determines mass distribution, prompt neutron evaporation and charge distribution, as shown below.

Mass Distribution

The familiar double humped mass yield curve of initial fragments (fission products corrected for prompt neutron evaporation) can be replaced in the light of the present model by a plot of the probability P of scission along the neck (Fig. 2). The scission point in Fig. 2 is described by its distance (d, in nucleons) from the heavy sphere. The prediction that can be drawn is that fission reactions yielding fragments with mass numbers outside the regions 132 to 154 and 82 to 104 should be unlikely and that a steep increase in mass yield should be found for the complementary fragment pairs $A_h' = 132$, $A_l' = 104$ and $A_l' = 82$, $A_h' = 154$ ($d = 0$ and 22).

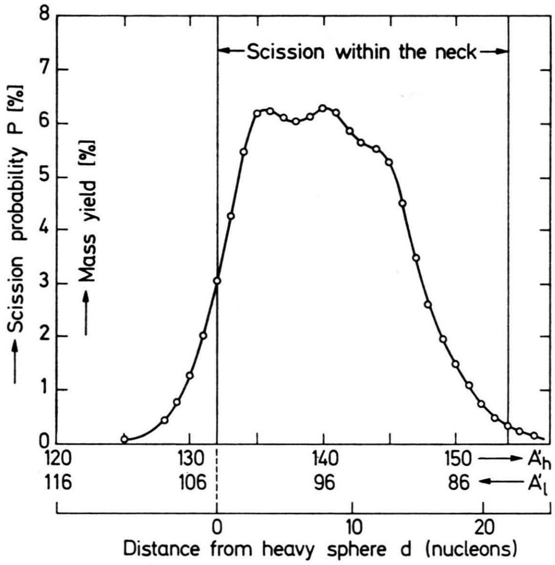

Fig. 2. Scission probability at different neck positions in the postulated model — an interpretation of the mass yield curve of initial fission fragments in ^{235}U (n_{th}, F) from [4]. (Light and heavy masses are complementary.)

Experimentally one finds that the yield of the fragments outside the said regions is $<10\%$ and that the increase in mass yield at $A' = 132$, 104 is steep as demonstrated in Fig. 2. The inflection point of the rising probability curve coincides with $d = 0$ not only in the fission of ^{235}U, as shown in the figure, but also in all low energy fission reactions in that mass range (see initial mass yield curves given by SCHMITT [9]). The rise of P at $d = 0$ may be even steeper than seen in Fig. 2 because the displayed curve is probably smoothed out by the finite resolution in measuring fragment mass yields. The decrease in fission yield following the maximum between $d = 2$ and $d = 10$ is shallower than the rise, starts earlier, and ends in a lower value of P at the end of the neck ($d = 22$) as compared to that at its onset ($d = 0$). This can be rationalized by the higher deformability of the smaller sphere [10] as compared

[7] A. C. WAHL, A. E. NORRIS, R. A. ROUSE, and J. C. WILLIAMS, 2nd Symp. on Physics and Chemistry of Fission, IAEA, Vienna, July 1969, Contribution SM-122/116.

[8] E. E. MASLIN, A. C. RODGERS, and W. G. F. CORE, Phys. Rev. **164**, 1520 [1967].

[9] H. W. SCHMITT, 2nd Symp. on Physics and Chemistry of Fission, IAEA, Vienna, July 1969, Contribution SM-122/122.

to that of the double magic larger one, resulting in an asymmetric neck thickness as indicated in Fig. 1.

Prompt Neutron Evaporation

The original concept of a dumb-bell shape of the fissioning nucleus was developed from the sawtooth prompt neutron emission curve [1, 2], interpreted as a consequence of internal excitation (= deformation) energy of the fragments. The model presented here with the more detailed assumptions on cluster and neck size allows some specified predictions: If the number of prompt neutrons is a consequence of deformation, then fragments with $A' = 132$ or 82 should emit the least neutrons because of their spheric shape, whereas the strongly deformed complementary fragments with $A' = 104$ or 154 should show strong neutron emission. Prompt neutron emission should increase with neck length and should be approximately independent of whether the neck is attached to the heavy or light sphere. It should also increase in the rare events when nucleons are broken out of a spheric structure. These predictions can be compared with experiment.

Prompt neutron emission has been redetermined recently using two independent methods [7, 8] with almost identical results. The data of MASLIN et al. [8] are shown in Fig. 3. The number of neutrons emitted from single fragments is plotted versus the length of the neck attached to the observed fragment as deduced from its mass using the present model. Neutron emission shows a pronounced minimum around $A' = 132$ and 82 ($d = 0$). A steady increase in neutron yield is found with increasing fragment mass, identical for both, heavy and light, fragments (blank and full points, respectively). The increase is also seen toward lower masses, i.e. when the spheric structure is destroyed by loss of nucleons. This argument may, however, be complicated by the appearance of a second, symmetric fission mode. A comparison with WAHL's data [7] (not shown) gives the same overall picture. Some fine structure seen may allow additional insight after further experimental confirmation.

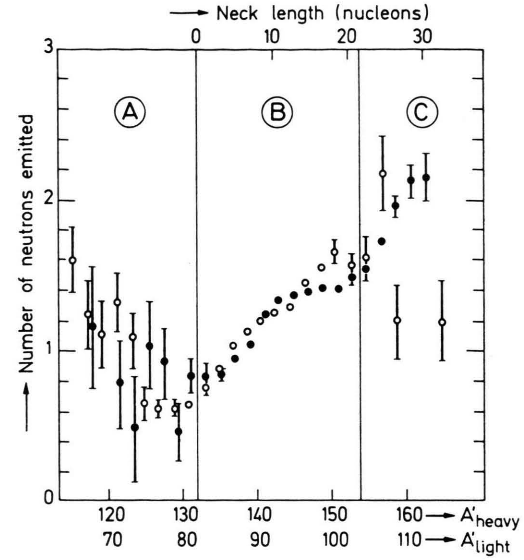

Fig. 3. Neutron evaporation from heavy (blank circles) and light fragments (full circles) in ^{235}U (n_{th}, F) as a function of scission point in the postulated dumb-bell configuration. *Section A:* Scission destroys sphere in observed fragment. *Section B:* Neutron emission proportional to increasing neck length. *Section C:* Scission destroys sphere in opposite fragment. Data from MASLIN et al. [8].

Charge Distribution

Charge distribution is predicted unambiguously by the present model for the two fragment mass pairs with $A_h' = 132$, $A_l' = 104$ and $A_l' = 82$, $A_h' = 154$, representing scission at the junction of neck and heavy or light sphere, respectively (Fig. 1). A linear interpolation of these two extreme cases, for scission anywhere else in the neck, gives our prediction for Z_p. It can be compared with experimental data in Fig. 4, in which, using WAHL's method [11], most probable charges (Z_p-values) are plotted after subtracting Z_p-values calculated assuming unchanged charge density (UCD). Negative deviation from 0 (= UCD) on the ordinate reflects higher neutron density in the heavy fragment and a correspondingly lower value in the light fragment. Experimental values [12] (points) and our prediction (straight bold line) are given. Several features are salient from the figure:

10 R. VANDENBOSCH, Nucl. Phys. **46**, 129 [1963].

11 A. C. WAHL, R. L. FERGUSON, D. R. NETHAWAY, D. E. TROUTNER, and K. WOLFSBERG, Phys. Rev. **126**, 1112 [1962].

12 Experimental independent yields compiled by WAHL et al. [7] were used to calculate Z_p-values on the assumption of a Gaussian charge dispersion curve with a width parameter $\sigma = 0.59 \pm 0.06$ where no experimental value of σ was available. Prompt neutron evaporation data of WAHL et al. [7] were adopted. Calculation with the data of MASLIN et al. [8] gives almost identical results.

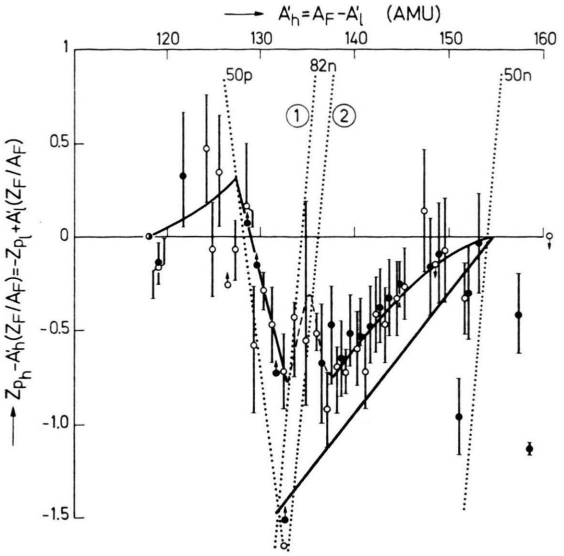

Fig. 4. Charge distribution curve for ^{235}U (n_{th}, F). Fragment mass numbers are calculated using prompt neutron evaporation data of WAHL et al. [7]. *Straight bold line:* prediction from present model. *Curved line:* smooth line drawn through experimental points, heavy fragment (blank circles), light fragment (full circles) — broken part around $N=82$ shows a fine structure. *Dotted lines:* nuclear shells (1) of fission fragments, (2) of fission products (after neutron evaporation) or of both.

The slope of the predicted line follows very closely that of a smooth line drawn through the experimental points — in contrast to all predictions from other fission postulates, like Equal Charge Displacement (ECD), Minimal Potential Energy (MPE) (see [11]) and — to a minor extent — the cluster model [6]. The deviation found in the present case in the region around $A'=134$ will be discussed later. A certain apparent curvature of the experimental points toward $Z_p - A'(Z_F/A_F) = 0$ can be interpreted as a tendency of the neutron-rich heavy fragment to collect preferentially protons from the neck.

The region of validity of the present model (straight bold line) is characterized by a steady and smooth variation of the experimental Z_p-values, whereas abrupt changes and large scatters are found in the area coinciding with the mass regions where the postulated spheric structures are broken in the fission process.

A fine structure (broken line) appears around $A'=134$. It could be interpreted as a consequence of the closed 82 neutron shell [13] and may have two causes: a primary effect on scission, producing preferentially products with $N=82$ just above $Z=50$. Such a process should affect also the complementary fission product, as has been observed with the fragment pair $A_h'=136$, $A_l'=98$ in the reaction ^{233}U(n_{th}, F) [14]. Probably a secondary effect contributes also: the feeding of nuclei with $N=82$ by prompt neutron emission from heavier precursors is enhanced as a consequence of the jump in neutron binding energy at the closed shell, an argument used also to explain the fine structure of the post neutron emission mass yield curve at mass number 134 [4].

Concluding, we would like to suggest a more detailed theoretical consideration of the presented picture.

We wish to thank Professor G. HERRMANN for encouragement and helpful discussions and Professor A. C. WAHL for making his data on prompt neutron evaporation and his compilation of fractional fission yields available to us prior to publication.

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